ITCS 314: Automata and Formal Languages

Exam 2, First semester 2014/2015, Form: A

Name: KEY

Student Number: _____

Section: _____

Section 1. (1 point each)

Mark the following statements with **True** if they are true and **False** otherwise.

F A grammar G is ambiguous if there exists a left-most derivation and a right-most derivation for the same word using G.

F Removing unit productions can produce λ-productions.

T The language $L = \{a^n b^n c^m\} \cup \{a^m b^n c^n\}$ is inherently ambiguous.

T In order for brute force search of a word in a language of a grammar to be an algorithm, the grammar must not contain λ or unit productions.

T The NPDA uses nondeterminism to accept languages such as $\{ww^R\}$.

T The language $L = \{a^n b^m : 2n \geq 7m\}$ is context-free.

F Every context free grammar can be converted to Greibach Normal Form.

T A variable A in a context-free grammar is useful if $S \overset{*}{\Rightarrow} xAy \overset{*}{\Rightarrow} w$ for some $w \in L$.

T The language $L = \{a^n b^m : n < m < 50n\}$ can be accepted by an NPDA.

T The grammar $S \to aSb|ab$ is equivalent to $S \to aAb, A \to aAb|\lambda$.

Section 2. (5 points each)

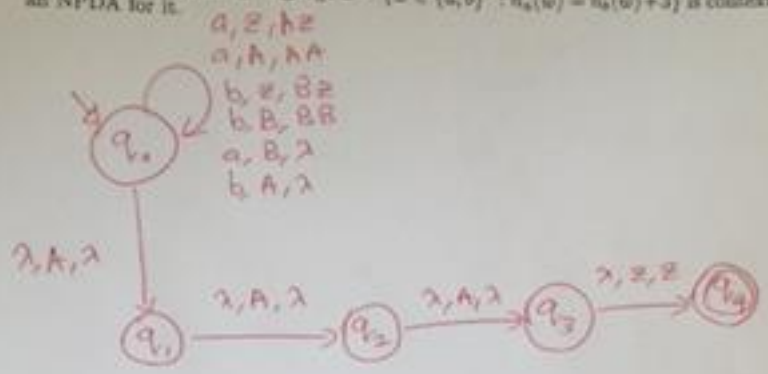
1. Construct a context-free grammar for the following language $L = \{a^n b^m a^k : n + k = 2m\}$.

$S \to AB \mid aAbBa$

$A \to aaAb \mid \lambda$

$B \to bBaa \mid \lambda$

2. Show that the following language $L = \{w \in \{a,b\}^* : n_a(w) = n_b(w)+3\}$ is context-free by constructing an NPDA for it.



Another solution:
Construct CFG → Convert to GNF → Convert to NPDA

3. Convert the following context-free grammar to an NPDA. Show your work.

$$S \rightarrow AB|bB$$
$$A \rightarrow b|aAb$$
$$B \rightarrow a$$

Remove λ and unit productions → None
Replace A in 1st production to allow conversion to GNF

$$S \rightarrow bB|aAbB|bbB$$
$$A \rightarrow b|aAb$$
$$B \rightarrow a$$

Convert to GNF:

$$S \rightarrow bB|aACB|bCB$$
$$A \rightarrow b|aAC$$
$$B \rightarrow a$$
$$C \rightarrow b$$

Convert to NPDA

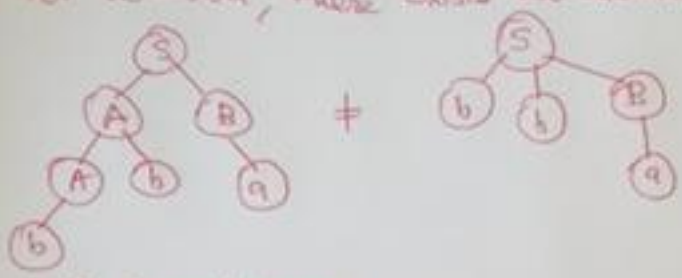
$\delta(q_0, \lambda, z) = \{(q_1, Sz)\}$
$\delta(q_1, \lambda, z) = \{(q_f, z)\}$

$\delta(q_1, b, S) = \{(q_1, B), (q_1, CB)\}$
$\delta(q_1, a, S) = \{(q_1, ACB)\}$
$\delta(q_1, b, A) = \{(q_1, \lambda)\}$
$\delta(q_1, a, A) = \{(q_1, AC)\}$
$\delta(q_1, a, B) = \{(q_1, \lambda)\}$
$\delta(q_1, b, C) = \{(q_1, \lambda)\}$

4. Show that the following grammar is ambiguous.

$S \rightarrow AB|bbB$
$A \rightarrow b|Ab$
$B \rightarrow a$

For $w = bba$, there exists two distinct derivation trees.



∴ G is ambiguous.

5. Remove the unit, useless, and λ-productions in the following grammar. Show your work.

$S \rightarrow aaSb|ABC|D$
$A \rightarrow b|Ab|\lambda$
$B \rightarrow a|aB$
$C \rightarrow aCb|a$
$D \rightarrow aDb|aD$

① Remove λ- productions

$V_N = \{A\}$
$S \rightarrow aaSb \mid ABC \mid BC \mid D$
$A \rightarrow b \mid Ab$
$B \rightarrow a \mid aB$
$C \rightarrow aCb \mid a$
$D \rightarrow aDb \mid aD$

② Remove unit-productions

$S \Rightarrow D$
$S \rightarrow aaSb \mid ABC \mid BC \mid aDb \mid aD$
$A \rightarrow b \mid Ab$
$B \rightarrow a \mid aB$
$C \rightarrow aCb \mid a$
$D \rightarrow aDb \mid aD$

③ Remove useless productions

D is useless
$S \rightarrow aaSb \mid ABC \mid BC$
$A \rightarrow b \mid Ab$
$B \rightarrow a \mid aB$
$C \rightarrow aCb \mid a$